RHEOLOGICAL DYNAMICS OF THE ELONGATIONAL FLOW OF A PLANE POLYMER STREAM DURING FILM DRAWING

V. M. Shapovalov and N. V. Tyabin

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The drawing of a flat stream in the formation zone is analyzed for isothermal conditions. Theoretical and experimental results are compared.

Uniaxial longitudinal flow usually takes place in the drawing of a flat film of plastic [1, 2]. However, experimental studies show that the uniaxial flow of the polymer may be disrupted when the film being formed is taken up on a cooling drum, and biaxial flow may occur. This phenomenon has a certain effect on the dimensions of the resulting film. Also, analysis of the drawing of a plane stream in the formation zone may be used to study the rheological properties of the polymer under conditions of biaxial drawing [3] and the process of heat exchange in the formation zone [4].

Presented below are the results of an analysis of the biaxial drawing of a plane stream in the formation zone, with allowance for the effect of the dimensions of the zone on the character of flow.

Figure 1 shows a diagram of the process of formation of the flat film. The drawing zone is located between the point of maximum distension and the point of contact of the film with the cooling drum. The rectangular coordinates at point $P_1$ of the film surface were chosen as follows: $\varepsilon_1$ corresponds to the direction of flow, $\varepsilon_2$ corresponds to the normal to the film surface, and $\varepsilon_3$ corresponds to the direction across the flow. The origin of the stationary coordinate system x, b, $\delta$ was placed at the point of greatest distension. The x axis is directed along the film. The quantities b and $\delta$ characterize the current width and thickness of the film.

The strain rate tensor may be written as follows:

$$|d| = \begin{vmatrix} d_{11} & 0 & 0 \\ 0 & d_{22} & 0 \\ 0 & 0 & d_{33} \end{vmatrix}, \tag{1}$$

where $d_{11} = \partial v_1/\partial \varepsilon_1$; $d_{22} = \partial v_2/\partial \varepsilon_2$; $d_{33} = \partial v_3/\partial \varepsilon_3$. It has been established [3] that $d_{22}$ and $d_{33}$ may be represented as functions of b, $\delta$, and x in the form

$$d_{22} = \frac{Q\delta'}{b\delta^2},\ d_{33} = \frac{Qb'}{b^2\delta}, \tag{2}$$

where $\delta' = \frac{d\delta}{dx}$; $b' = \frac{db}{dx}$; $Q = vb\delta$. From the condition of continuity $d_{11} + d_{22} + d_{33} = 0$ we find that

$$d_{11} = -\frac{Q}{b\delta}\left(\frac{b'}{b} + \frac{\delta'}{\delta}\right). \tag{3}$$

The stress components $\sigma_{ij}$ may be written in general form as follows:

$$\sigma_{ij} = -p\delta_{ij} + \tau_{ij}\ (i,\ j = 1,\ 2,\ 3), \tag{4}$$

where $\tau_{ij}$ is the stress tensor deviator.

At a stress on the free surface equal to the atmospheric pressure, the following condition follows from Eq. (4)

$$\sigma_{22} = 0. \tag{5}$$

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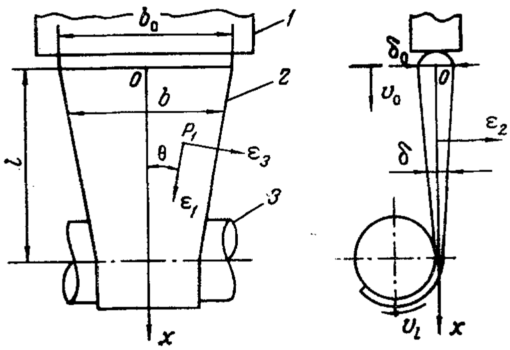


Fig. 1. Process of formation of flat film: 1) draw plate; 2) polymer stream; 3) cooling drum.

Substituting (5) into (4), we obtain

$$p = \tau_{22}. \tag{6}$$

The longitudinal viscosity of the melt η may be determined thus

$$\tau_{ij} = \eta\left[\left(\frac{I_2}{2}\right)^{1/2}\right] d_{ij}\ (i,\ j = 1,\ 2,\ 3)\,, \tag{7}$$

where

$$\left(\frac{I_2}{2}\right)^{1/2} = (d_{11}^2 + d_{22}^2 + d_{33}^2)^{1/2}.$$

We are assuming that the viscosity is independent of temperature. Substituting (6) into (4), we obtain

$$\sigma_{11} = \tau_{11} - \tau_{22},\ \sigma_{33} = \tau_{33} - \tau_{22}$$

or, allowing for (7),

$$\sigma_{11} = \eta\left[\left(\frac{I_2}{2}\right)^{1/2}\right](d_{11} - d_{22}), \tag{8}$$

$$\sigma_{33} = \eta\left[\left(\frac{I_2}{2}\right)^{1/2}\right](d_{33} - d_{22}). \tag{9}$$

It was observed in experiments that, in the absence of drawing, the width of the film does not change and there is no sagging. Thus, the forces due to the weight of the film and surface tension are negligibly small. The balance of the forces acting on the film are as follows to a first approximation

$$\sigma_{11} b\delta = F. \tag{10}$$

The inertial force is ignored. Expression (10) can be used to experimentally study the effect of effective longitudinal viscosity, entering into (8) and (9), on the strain rate [3]. In the case

$$\sigma_{33} = 0 \tag{11}$$

a regime of uniaxial drawing is maintained. In this case, the cross sections of the film have geometric similitude [1, 2].

It was experimentally established that this similitude is disturbed in the direction of an increase in film width when $l/b_0$ decreases, i.e., condition (11) does not hold in this case. If we examine the edge of the film as a filament in tension, we should expect that this filament will strive toward a position in which it covers the shortest distance, i.e., when the winding plane is perpendicular to the drum rotation axis. The forces associated with the viscosity of the material will oppose such a displacement of the edge. It may be assumed that these forces will be proportional across and along the film, i.e.,

$$\sigma_{33} = K_1\sigma_{11}, \tag{12}$$

where $K_1$ is a coefficient dependent on the dimensions of the formation zone.

Allowing for (10) and (12), the balance of forces over the width of the film will be

$$\sigma_{33} b\delta = K_1 F. \tag{13}$$

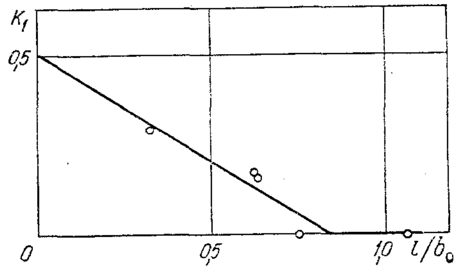


Fig. 2

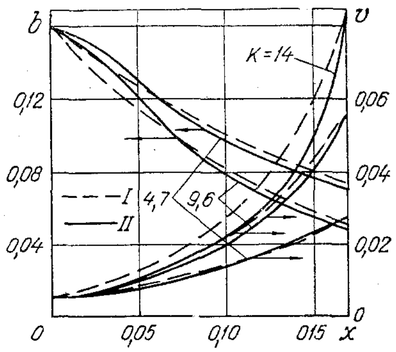


Fig. 3

Fig. 2. Dependence of $K_1$ on dimensions of the formation zone.

Fig. 3. Distribution of velocity and width of film along the formation zone: I) calculation; II) experiment. v, m/sec; b, m; x, m.

Substituting Eqs. (2), (3), (8), and (9) into (10) and (13), we obtain a system of differential equations

$$\frac{b'}{b}+\frac{2\delta'}{\delta}=-a,\quad \frac{b'}{b}-\frac{\delta'}{\delta}=aK_1,$$

the solutions of which have the following form for the initial condition $x=0$, $\delta=\delta_0$, $b=b_0$ and the condition $\eta=\text{const}$

$$b=b_0\exp\left[\frac{a}{3}(2K_1-1)x\right], \tag{14}$$

$$\delta=\delta_0\exp\left[-\frac{a}{3}(K_1+1)x\right], \tag{15}$$

where $a=F/\eta Q$.

It follows from an analysis of (14) and (15) that uniaxial drawing takes place at $K_1=0$ and biaxial drawing takes place at $K_1=0.5$. Allowing for (14) and (15), we may obtain the following expression for longitudinal velocity from the continuity equation $Q=vb\delta$

$$v=v_0\exp\left[\frac{a}{3}(2-K_1)x\right], \tag{16}$$

where $v_0=Q/b_0\delta_0$.

Experimental determination of the tensile force F is very difficult, so that it is convenient to use the condition $v|_{x=l}=v_l$ in order to exclude $a$ from (14), (15), and (16). Here, for the width and velocity we have

$$b=b_0\exp\left[\frac{(2K_1-1)x}{(2-K_1)l}\ln K\right], \tag{17}$$

$$v=v_0K^{x/l}. \tag{18}$$

After analyzing the experimental results, we constructed a graph of the dependence of $K_1$ on the dimensions of the formation zone (Fig. 2). On the section $0 \ll l/b_0 \ll 0.85$, the relation is approximated by the expression

$$K_1=0.5-0.588\frac{l}{b_0}. \tag{19}$$

The conditions of formation of the flat polypropylene film were varied within the following ranges during the experiments: $K=1.4$–$16$; $b_0=0.16$–$0.278$ m; $l=0.09$–$0.17$ m; $v_0=5\cdot10^{-3}$–$16.7\cdot10^{-3}$ m/sec, $Q=1.5\cdot10^{-6}$–$5\cdot10^{-6}$ $m^3$/sec. The gap between the jaws of the draw plate was 0.5 mm. The temperature of the draw plate was 250°C. The drop in the temperature of the film by the end of drawing reached 30°K, i.e., the film cooled to 220°C.

It follows from Fig. 2 that uniaxial drawing begins at $l/b_0 > 0.85$. To experimentally determine the relation $K_1 = K_1(l/b_0)$, it is sufficient to measure the initial and final widths of the film for different drawing-zone dimensions. In this case, it is necessary to set $x = l$ in theoretical formula (17). In forming polymer films of other polymers (or in another temperature interval), it may be expected that the character of $K_1 = K_1(l/b_0)$ will be different from that depicted in Fig. 2 since the flow of the melt in the formation zone depends on its rheological properties.

The method of marking was used to measure the axial-velocity distribution. We also measured the film profile at different drawing speeds. The conditions of formation of the polypropylene film were as follows: $T = 250°C$; $Q = 1.16 \cdot 10^{-6}$ m$^3$/sec; $b_0 = 0.16$ m; $K_1 = 0$. The gap between the jaws of the die plate was 0.5 mm. Figure 3 compares the theoretical and experimental results. The discrepancy between the calculated and experimental data can be attributed to the effect of inertial forces (particularly at high drawing speeds) and the effects of viscosity anomaly and variable temperature. It is interesting to note that, in the above formulation, the distribution of axial velocity in the formation zone (18) is independent of the rheological properties of the polymer.

The completed studies of film drawing can also be used in analyzing oriented drawing on rolls, where the question of change in film width is important [5-7].

## NOTATION

$b_0$, $\delta_0$, initial width and thickness of film; $l$, length of formation zone; $\delta$, b, current thickness and width of film; x, longitudinal coordinate; $v_0$, axial velocity in the distended section; v, current axial velocity; $v_l$, drawing speed; $\varepsilon_1$, $\varepsilon_2$, $\varepsilon_3$, convective coordinates; $\theta$, angle between the $\varepsilon_1$ and x axes; $|d|$, strain rate tensor; $d_{ij}$, strain rate components; $v_1$, $v_2$, $v_3$, velocity components in the convective coordinate system; Q, volumetric flow rate; $\sigma_{ij}$, stress components; p, isotropic pressure; $\delta_{ij}$, Kronecker delta; $\tau_{ij}$, stress tensor deviator; $\eta$, viscosity; $I_2$, second invariant of strain rate tensor; F, tensile force; $K = v_1/v_0$, actual multiplicity factor of draw-plate elongation; T, temperature.

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